MOTION ANALYSE FOR A VIBRATORY SEPARATING MACHINE CONCERNING STIFFNESS AND DAMPING OF RUBBER SUPPORTING ELEMENTS

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***Abstract*—**This work paper refers to a specific type of machines functioning known as granular materials separator, which are using vibratory motion for their working elements (sieves). An efficient vibratory motion for this type of machines produces a process of self-sorting for granules of different masses and sizes in the nearby surface of the sieve. Therefore mechanical systems of separating machines are using an adequate supporting system and a specified generative force system in order to produce a certain trajectory for the center of mass of the working element (Lissajous elliptic type curves). For this not only stiffness constants (influenced by material characteristics and shape) of supporting elements but also damping constants is a very important factor that influence motion.

Keywords — damping, stiffness, supporting system elements, vibratory motion trajectory.

I. INTRODUCTION

THIS is a theoretical for-presentation in order to understand some aspects concerning dynamic of a vibratory machine, and its relevant parameters. Vibratory machines as they are presented in [1], [4], [6] are many types, concerning their constructive purpose, their supporting systems, their vibration (excitation force) generating system, etc. This paper is about a separating-classifier type vibratory machine. Its mechanical system, is known as directional vibrations generating system machine whose principal parts are shown in Fig. 1, [7].

Generalized mathematical model is given by Eq. (1), as given in [1] – [3]:

$$\begin{aligned}(\mathbf{m}+\mathbf{m_0})\ddot{\mathbf{x}}+\mathbf{c_x}\dot{\mathbf{x}}+\mathbf{k_x}\mathbf{x}&=\mathbf{m_0}\mathbf{e}\omega^2\cos\alpha\cdot\cos\omega\mathbf{t}\\(\mathbf{m}+\mathbf{m_0})\ddot{\mathbf{z}}+\mathbf{c_x}\dot{\mathbf{z}}+\mathbf{k_z}\mathbf{z}&=\mathbf{m_0}\mathbf{e}\omega^2\sin\alpha\cdot\cos\omega\mathbf{t}\end{aligned} \tag{1}$$

where: $m+m_0$ is total mass of the vibrating system, while m_0 is non-equilibrated mass of the generator; $k_{x,z}$ stiffness coefficients, and $c_{x,z}$ damping coefficients in Ox and Oz direction of supporting elements, e is radius of the center of mass position of non-equilibrated mass of generators, ω is angular velocity of generators motors and therefore pulsation of generative centrifugal force which is directed constantly at α angle to horizontal direction Oz as it is presented in Fig. 2.

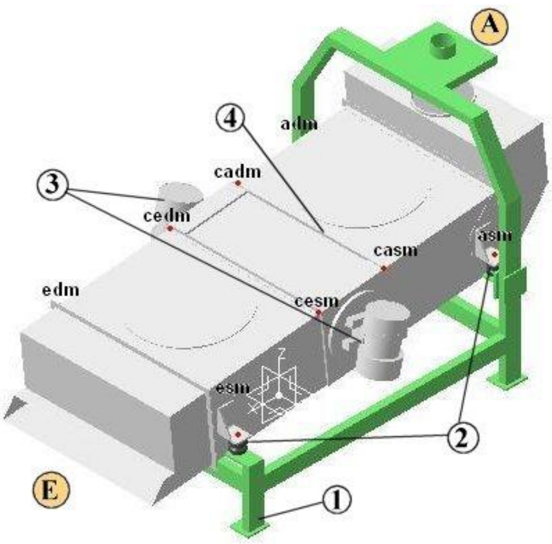


Fig. 1. A 3D model (CATIA©) for a separator, a directional vibration generating mechanical system structure: 1 – supporting steel structure; 2 – supporting elastic-damping elements (rubber); 3 – vibration force generators (non-equilibrated masses);4 – working element (sieves system); A – admission for granular material; E – evacuation for separated granular material (non-refused)

Solution of differential equations system (1) is:

$$\mathbf{x}=\mathbf{x_a}\cdot\cos\left(\omega\mathbf{t}+\phi_\mathbf{x}\right) \quad ; \quad \mathbf{z}=\mathbf{z_a}\cdot\cos\left(\omega\mathbf{t}+\phi_\mathbf{z}\right) \tag{2}$$

where, parameters are:

$$x_a = \frac{m_0 e \omega^2 \cos\alpha}{(m+m_0)\sqrt{\left(p_x^2-\omega^2\right)^2 + 4n_x^2\omega^2}};$$
$$y_a = \frac{m_0 e \omega^2 \sin\alpha}{(m+m_0)\sqrt{\left(p_y^2-\omega^2\right)^2 + 4n_y^2\omega^2}}, \qquad (3)$$

$$tg\phi_x = \frac{2n_x\omega}{p_x^2-\omega^2} \quad ; \quad tg\phi_z = \frac{2n_z\omega}{p_z^2-\omega^2} \qquad (4)$$

$$p_{x,z} = \sqrt{\frac{k_{x,z}}{m+m_0}} \quad ; \quad n_{x,z} = \frac{c_{x,z}}{2(m+m_0)} \qquad (5)$$

The two functions in Eq. (2) are parametric description of an elliptic Lissajous curve showed in Fig. 2, which has the form given by Eq. (6), [4], [5]:

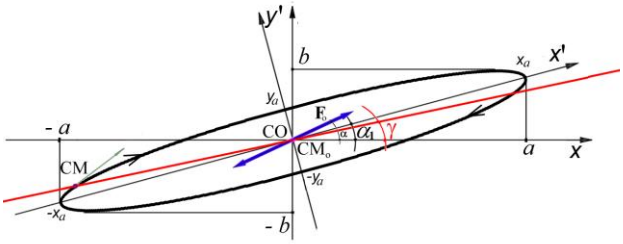


Fig. 2. Center of Mass (CM) trajectory form around Center of Oscillation (CO): γ is the inclination angle for sieves plane from evacuation (E) to admission (A) in Fig. 1.

$$\frac{x^2}{x_a^2} + \frac{z^2}{z_a^2} - \frac{2xz}{x_a z_a}\cos(\phi_x-\phi_z) = \sin^2(\phi_x-\phi_z) \qquad (6)$$

for which:

$$tg2\alpha_1 = \frac{2x_a z_a \cos(\phi_x-\phi_z)}{x_a^2 - z_a^2} \qquad (7)$$

$$a = \frac{x_a z_a \left|\sin(\phi_x-\phi_z)\right|}{\sqrt{\frac{1}{2}(x_a^2+z_a^2) - \frac{1}{2}(x_a^2-z_a^2)\cos 2\alpha_1 - x_a z_a \cos(\phi_x-\phi_z)\sin 2\alpha_1}}; \qquad (8)$$

$$b = \frac{x_a z_a \left|\sin(\phi_x-\phi_z)\right|}{\sqrt{\frac{1}{2}(x_a^2+z_a^2) + \frac{1}{2}(x_a^2-z_a^2)\cos 2\alpha_1 + x_a z_a \cos(\phi_x-\phi_z)\sin 2\alpha_1}}; \qquad (9)$$

II. MODELING AND PARTICULARITIES

A model in Matlab with Simulink R2009b© has been created in order to study parameters that influence trajectory described in Equations (3) to (9).

In Fig. 3 and 4 is presented the block diagram using block from SimMechanics Toolbox:

Main parameters of model presented in Fig. 3 and 4 are:

$$m = 367{,}5\ (kg)\ ;\ m_0 = 26{,}52\ (kg)\ ;\ e = 50{,}017\ (mm)$$
$$k_x = k_z = 5.10^5\ (N/m)\ ;\ c_x = c_z = 10^4\ (N.s/m)\ ;$$
$$k_{ry} = 2.10^6\ (N.m/m)\ ;\ c_{ry} = 10^5\ (Nm.s/m).$$

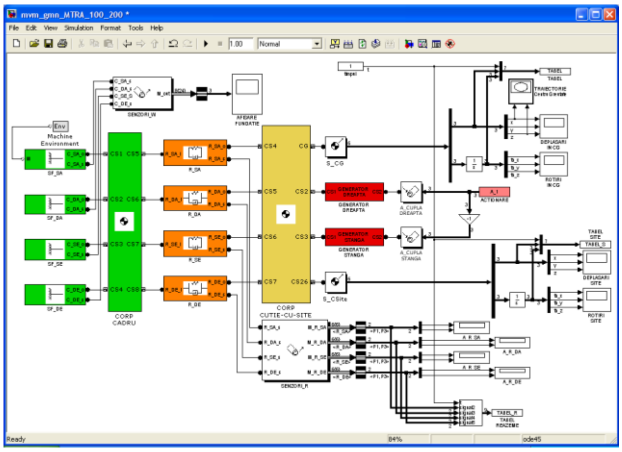
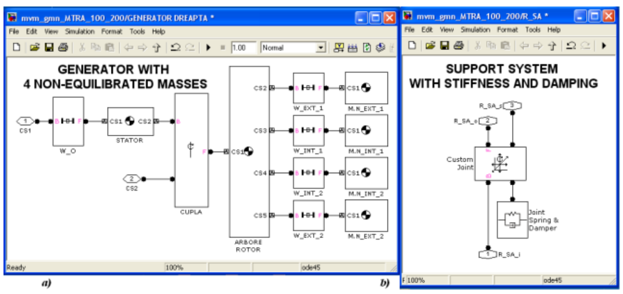


Fig. 3. Block diagram of the model of directional vibration generating mechanical system.

Model in Fig. 3 use for modeling rubber supports stiffness and damping, a specific block of Matlab with Simulink R2009b©, named <<Joint Spring & Damper>>, attached to another specific block named << Custom Joint >> as seen in Fig. 4 b). This technique is used because the already known, number of degrees of freedom for support, seen as Joint, are altered with restriction of motion given by stiffness and damping in a [8], [9]. In Fig. 4 a) it is presented model for the generator with four non equilibrated masses which uses another specific block named <<Weld>>, connecting blocks like <<Body>> in various angular positions in order to set variable values for center of mass position and therefore variable centrifugal force for excitation.

Block like <<Joint Spring & Damper>> are using feed-back circuit techiques to model visco-elastic reaction forces in supports [9].



a) b)

Fig. 4. *a)* Block diagram for generator with non-equilibrated masses; *b)* Block diagram for support system with stiffness and damping

This model permits us to give a proper appreciation of the real behavior of the mechanical system in discussion. Therefore we have to mention some important factors:

– Natural pulsations $p_{x,z}$ in correlation to directional generating force pulsation ω, [1] : in absence of damping ($c_x = c_z = 0$) in for-resonant ($\alpha_1 < \alpha$) or post-resonant ($\alpha_1 > \alpha$) trajectories are in the same quadrant with force;
– If natural pulsations are equal $p_x = p_z$ then, also in absence of damping, center of mass (CM) trajectory has the same angle with generating force, $\alpha_1 = \alpha$. That implies also $k_x = k_z$ characteristic for supporting elements 2 in Fig. 1 (same stiffness for bending and compression).

For this, a certain shape of supporting element was taken in consideration, as shown in Fig. 5, shape that gives correlation with aspects shown above.

As we may see, shape in Oxz plane of has its exterior and interior aspect of a bending and compression-tension stiffness modulus equal one to each other.

Also it is to note that different required shape aspects may be required for different elasticity characteristics valued and therefore different trajectories aspects

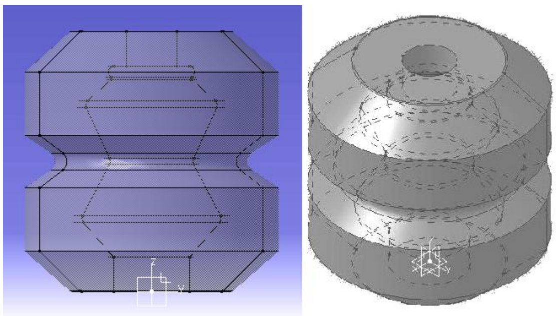


Fig. 5. High Damping Rubber supporting elements characteristic shape. Same stiffness for bending and compression

In presence of a certain damping, correlation between α_1 angle and α angle cannot be predicted in the same way. Not only supporting elements, their shape and material characteristics are important this time, but also a variable damping characteristic of the granular material on the sieve.

Therefore a trajectory analysis has been performed. In Fig. 6, bellow, trajectories for different points are shown. In simulation diagram in Fig. 3 test point (CS26) has variable coordinates, with respect of center of mass (CG) as shown in Table I. All trajectories in Fig. 6 are reduced to working element center of gravity (CG) in model in Fig. 3.

Each trajectory in Fig. 6, have different parameters and correspond to different vibratory motions of the points coordinates nominated in Table I. There, one may see causes for sieving and separation of granular material defects.

TABLE I
COORDINATES FOR POINT CONSIDERED IN CALCULUS

Name on block diagram	x (mm	y (mm)	z (mm)	Remarks
CS4	-863	0	183,406	Admission
CS26_4	-514	0	-69,9558	On sieve 4
CS26_3	-165	0	-33,2744	On sieve 3
CG	0	0	670	Center of mass
CS26_2	184	0	-106,6372	On sieve 2
CS26_1	533	0	-143.3186	On sieve 1
CS6	882	0	-180	Evacuation

For example a non-adequate throwing angle may cause during the process presence of what should be rejected granular material in the mass of good sieved granular material.

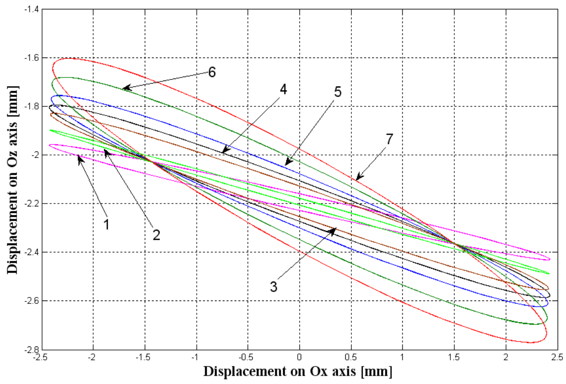


Fig. 6. Comparative graphics for trajectories aspects in different points on working element of machine (see Table I): 1 – Admission support point (CS4); 2 – on sieve point 4 (CS26_4); 3 - on sieve point (CS26_3); 4 – center of mass (CG); 5 - on sieve point CS26_2; 6 - on sieve point CS26_1; 7 – evacuation support point CS6

As shown in Fig. 6, with respect to the same reference point center of CG, different point on working element from admission to evacuation (CS26) have different trajectories and therefore different trajectory parameters (see Table II)

TABLE II
TRAJECTORIES CHARACTERISTICS EXAMPLES

Name on block diagram	a (mm)	b (mm)	α_1 (°)	Remarks
CS 4	2,426	0,2235	5,2636	Admission
CG	2,430	0,396	9,2557	Center of mass
CS6	2,393	0,5835	13,7034	Evacuation

In Fig. 7 is presented aspect of vibration motion on Oz axis for the last two periods of motion (the most significant of the two motions analyzed on Ox and Oz axis respectively).

From Table II and Fig. 7 we may conclude that optimal angle for trajectory (which gives optimal values of throwing angle) is for Center of Mass position point.

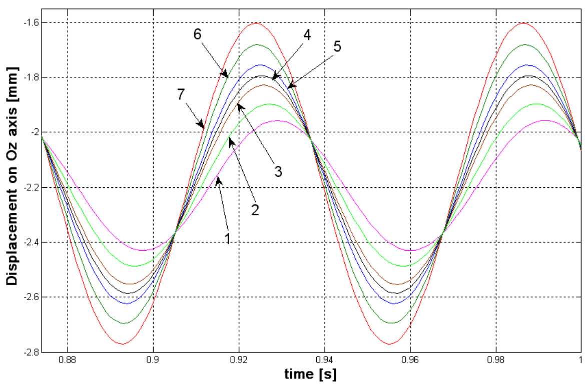


Fig. 7. Relative aspect of Oz vibration motion for different points on working element (CS4, CS26_4, CS26_3, CG, CS26_2, CS26_1, CS 6)

An elliptic type of Lissajous trajectory curve due to one of the characterization of vibratory motion analyzed, which is a planar two dimensional motion.

Elliptical form is caused for the reason that vibratory motions have the same pulsations (or frequencies) on Ox axis and on Oz axis, given by the generators centrifugal inertia forces components.

More, pulsations of generating forces are in for-resonant or post-resonant domain of functioning. This it is explained by same stiffness coefficients for bending and tension of supporting elements (depending by their shape).

A certain damping of supporting elements, but not only, influences motion trajectory ellipse position with respect to reference system Oxz. That means position angle depends on damping characterization of the system

Analyzing trajectories for different points on working element of the vibratory machine, one may observe different geometric characteristics from admission (A) to evacuation (E) supporting points passing in a rotating shape aspect through center of mass (CG).

Yet medium inclination angle of $\alpha_1 = 9{,}2557°$ for center of mass trajectory is appreciated as good for a $\gamma = 6{,}0°$ inclination of sieve plane and a $\alpha = 15{,}5°$ inclination of the rotation axes of generators.

III. Conclusion

It is considered as a good machine functioning (according to purpose that has been engineered for) a certain throwing angle realized in sieving motion of a certain correspondent separating process for a certain granular material.

Angles for trajectories obtained in Fig. 6 are relevant for functioning of the present analyzed model, with respect to certain values for stiffness and damping and, more, a certain type of process (separation) for a certain type of granular matter (wheat grains).

For a different type of process to sustain (like, classification for ex.) for a different type of granular material one has to use different engineered machine with different mechanical characteristics stiffness and especially damping.

It is desired an engineered type of machine that has to correlate all this factors in a supporting system with elements that can be adjusted easily to certain values of stiffness and especially damping in order to sustain different shape of trajectories of vibrating motion for different effective separation of granular material processes.

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